Comparison between theoretical predictions and direct numerical simulation results for a decaying turbulent suspension

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(Received 12 November 2003; published 26 May 2004)

A recently developed theoretical model for a turbulently flowing suspension has been applied to a homogeneous, isotropic, and decaying turbulent suspension. The predictions are compared with results from direct numerical simulations. The agreement is reasonable. Special attention is paid to a physical explanation of the influence of the particles on the turbulence of the carrier fluid.

DOI: 10.1103/PhysRevE.69.056311 PACS number(s): 47.27.−i, 47.55.Kf

I. INTRODUCTION

In the past much attention has been paid to the study of a turbulent fluid flow with particles. From these studies it is known that when the mass loading of the particles is considerable the two-way coupling effect of the fluid on the particles and vice versa must be taken into account. This two-way coupling effect has been studied by means of direct numerical simulations (DNSs), experiments, and theoretical models. Below a listing of some important publications about this topic is given. A more detailed description about their content can be found in the publication of L'vov, Ooms, and Pomyalov [1]. For an even more detailed overview we refer to the coming review article by Ooms and Poelma [2]. We have restricted ourselves to publications about homogeneous and isotropic, turbulently flowing suspensions, as the research presented in this publication also deals with such a suspension. We will pay special attention to the influence of the particles on the turbulent kinetic energy spectrum of the carrier fluid.

Several authors have applied DNS to particle-laden homogeneous, isotropic turbulent flows. For instance, Squires and Eaton [3] used DNS to study a forced (so statistically stationary) homogeneous, isotropic turbulent suspension. Elghobashi and Truesdell [4] examined turbulence modulation by particles in decaying turbulence. Similar DNS studies (for a stationary or decaying turbulent suspension) with more details were carried out by Boivin, Simonin, and Squires [5], Sundaram and Collins [6], Druzhinin [7], ten Cate [8], and Ferrante and Elghobashi [9]. From these studies it can be concluded that in the low wave number part of the turbulent kinetic energy spectrum the turbulent fluid motion transfers energy to the particles, i.e., the particles act as a sink of kinetic energy. At high wave numbers of the spectrum the particles (when their response time is small enough) are capable of adding kinetic energy to the turbulence. This energy, "released" by the particles, is not immediately dissipated by viscous effect but is in fact responsible for the relative increase of small-scale energy compared to the particle-free case.

Several articles about experimental investigations of particle-laden turbulent suspensions have been published. For instance, Tsuji and Morikawa [10] measured turbulent kinetic energy spectra in an air flow with small particles through a horizontal channel. Tsuji, Morikawa, and Shiomi [11] extended this work to a vertical channel flow. Schreck and Kleis [12] studied turbulence modulation by particles in grid-generated turbulence in a water channel. Kulick, Fessler, and Eaton [13] carried out experiments with small copper particles in an air flow through a channel flow. Sato and Hishida [14] performed PIV measurements in a water channel flow with three types of particles. Similar to Schreck and Kleis, Hussainov *et al.* [15] also investigated a grid-generated turbulent flow with particles, but they used a wind tunnel. Some of these experiments seem to support the conclusion given above with respect to the influence of the particles on the turbulence spectrum based on DNS simulations. However, more detailed experimental work is needed.

Also theoretical models have been developed for a homogeneous, isotropic, and turbulently flowing suspension. Baw and Peskin [16] derived a set of balance equations to study the effect of the particles and the turbulent kinetic energy spectrum of the fluid. Al Taweel and Landau [17] calculated the rate of additional energy dissipation due to the presence of the particles, in order to study the two-way coupling effect. Felderhof and Ooms [18] developed a theoretical model based on the linearized version of the Navier-Stokes equation and pay particular attention to the influence of the hydrodynamic interaction between the particles and the influence of the finite size of the particles. Yuan and Michaelides [19] presented a model for the turbulence modification in particle-laden flows based on the interaction between particles and turbulence eddies. Also the turbulence generation in the wake behind the particles was taken into account. Boivin, Simonin, and Squires [5] extended the model of Baw and Peskin. Druzhinin [7] studied the two-way coupling effect on the decay rate of isotropic turbulence laden with microparticles whose response time is much smaller than the Kolmogorov time scale.

Recently L'vov, Ooms, and Pomyalov [1] developed a one-fluid theoretical model for a homogeneous, isotropic turbulently flowing suspension. It is based on a modified Navier-Stokes equation with a wave-number-dependent effective density of suspension and an additional damping term representing the fluid-particle friction (described by Stokes' law). The statistical description of turbulence within the model is simplified by a modification of the usual closure procedure based on the Richardson-Kolmogorov picture of turbulence. A differential equation for the budget of turbulent kinetic energy is derived. For the case of a stationary turbulent suspension L'vov *et al.* solved this budget equation analytically for various important limiting cases and numerically for the general case. The model successfully explains observed features of numerical simulations and experimental results of stationary turbulent suspensions. For instance, for a suspension with particles with a response time much larger than the Kolmogorov time the main effect of the particles is suppression of the turbulence energy of fluid eddies of all sizes (at the same energy input as for the particle-free case). However, for a suspension with particles with a response time comparable to or smaller than the Kolmogorov time scale, the Kolmogorov length scale will decrease and the turbulence energy of eddies of (nearly) all sizes increases. For a suspension with particles with a response time in between the two limiting cases mentioned above the energy of the larger eddies is suppressed whereas the energy of the smaller eddies is enhanced. We think that the model of L'vov, Ooms, and Pomyalov gives a good description of the physical mechanisms taking place in a turbulent suspension. Therefore, we have extended their model in this publication.

In their paper L'vov *et al.* do not apply their model to a decaying turbulent suspension. Recently Ferrante and Elghobashi [9] published results concerning very accurate direct numerical simulations of a decaying, homogeneous, and isotropic turbulent suspension. In their work they again found the phenomena described above for the influence of the particles on the turbulence. (In some earlier publications these phenomena were already discussed for a decaying suspension, but with less detail and less attention to their physical explanation.) Therefore we decided to extend the theoretical model in such a way that it can be applied to a decaying, homogeneous, and isotropic turbulent suspension and to compare model predictions with the DNS data of Ferrante and Elghobashi. The results are given in this publication.

Section II is devoted to a brief summary of the one-fluid theoretical model, with attention for the extension which is necessary for the application to a decaying suspension. In Sec. III the relevant DNS results (and their explanation) given in the publication by Ferrante and Elghobashi [9] are summarized. In Sec. IV predictions made with our theoretical model for a decaying turbulent suspension and a comparison with the DNS results of Ferrante and Elghobashi are given. The results are discussed in Sec. V.

II. THEORETICAL MODEL

Starting from the Navier-Stokes equation for the carrier fluid and Stokes' friction law for the particles L'vov *et al.* [1] derive first the following equation of motion for the suspension:

$$\rho_{\mathrm{eff}}(k)\left(\frac{\partial}{\partial t}+\gamma_p(k)+\gamma_0(k)\right)\mathbf{u}(t,\mathbf{k})=-\mathbf{N}\{\mathbf{u},\mathbf{u}\}_{t,\mathbf{k}}+\mathbf{f}(t,\mathbf{k}). \tag{1}$$

In this equation $\mathbf{u}(t,\mathbf{k})$ represents the suspension velocity, t the time, and $\mathbf{k}$ the wave vector. $\mathbf{f}(t,\mathbf{k})$ is the stirring force that creates turbulence in case a stationary suspension is studied. $\rho_{\mathrm{eff}}(k)$ is the wave-number-dependent effective density of suspension given by

$$\rho_{\mathrm{eff}}(k)=\rho_f\left(1-\psi+\phi\frac{1+2\tau_p\gamma(k)}{[1+\tau_p\gamma(k)]^2}\right), \tag{2}$$

in which ρ_f is the density of the carrier fluid, ψ the volume fraction of the particles in the suspension, ϕ the mass fraction of the particles, τ_p the particle response time, and $\gamma(k)$ the frequency of a turbulent eddy with wave number k. $\gamma_p(k)$ is an additional damping term representing the fluid-particle friction (described by Stokes' law)

$$\gamma_p(k)=\frac{\phi\tau_p[\gamma(k)]^2}{(1+\phi)[1+2\tau_p\gamma(k)]+[\tau_p\gamma(k)]^2}. \tag{3}$$

The damping term $\gamma_0(k)$ is due to the internal friction within the carrier fluid and is given by

$$\gamma_0(k)=\nu_{\mathrm{eff}}(k)k^2, \tag{4}$$

with

$$\nu_{\mathrm{eff}}(k)=\nu\rho_f/\rho_{\mathrm{eff}}(k), \tag{5}$$

in which ν is the viscosity of the carrier fluid. $\mathbf{N}\{\mathbf{u},\mathbf{u}\}_{t,\mathbf{k}}$ is the nonlinear term. The explicit form that is derived for it in the publication of L'vov *et al.* is not given here. It is not required for the simple closure procedure that was used in the original publication and which is also used here. For the introduction of the energy flux in the used closure procedure it is enough to use the fact that the modeled nonlinearity must be conservative. (The explicit form for the nonlinear term is needed, however, for more advanced closure procedures.)

From Eq. (1) L'vov *et al.* derive for (a homogeneous and isotropic) turbulent suspension the following budget equation for the spectrum of the density of turbulent kinetic energy $E_s(t,k)$ of the suspension:

$$\frac{1}{2}\frac{\partial E_s(t,k)}{\partial t}+[\gamma_0(k)+\gamma_p(k)]E_s(t,k)=W(t,k)+R(t,k). \tag{6}$$

The left-hand side of this equation includes next to the time-dependent term two damping terms $\gamma_0(k)E_s(t,k)$ caused by the effective viscosity and $\gamma_p(k)E_s(t,k)$ caused by the fluid-particle friction. The right-hand side includes the source of energy $W(t,k)$ due to a possible stirring force (localized in the energy containing interval of the spectrum) and the energy redistribution term $R(t,k)$ due to the interaction between turbulence eddies. Using the assumption that the modeled nonlinearity is conservative the energy redistribution term can be written as

$$R(t,k)=-\frac{\partial\epsilon(t,k)}{\partial k} \tag{7}$$

in which $\epsilon(t,k)$ is the energy flux through the turbulence eddies of the suspension.

As mentioned above, a simple closure relation is used. Applying dimension analysis the following relation is found for the density of turbulent kinetic energy in terms of the energy flux

$$E_s(t,k) = C_1[\epsilon^2(t,k)\rho_{\mathrm{eff}}(t,k)/k^5]^{1/3}. \tag{8}$$

C_1 is a constant of order unity. The inverse lifetime (frequency) of eddies $\gamma(t,k)$ is determined by their viscous damping and by the energy loss in the cascade process

$$\gamma(t,k) = \gamma_0(t,k) + \gamma_c(t,k). \tag{9}$$

The inverse lifetime due to viscous damping has already been introduced by Eqs. (4) and (5). Applying dimension analysis the inverse lifetime of a k eddy due to energy loss in the cascade process is given by

$$\gamma_c(t,k) = C_2[k^2\epsilon(t,k)/\rho_{\mathrm{eff}}(t,k)]^{1/3}. \tag{10}$$

C_2 is again a constant of order unity.

After introducing the integral-scale (L) related parameters $\kappa = kL$, $\epsilon_L = \epsilon(1/L)$, $\gamma_L = \gamma(1/L)$, $\rho_L = \rho_{\mathrm{eff}}(1/L)$, and $W_L = W(1/L)$ the following dimensionless functions are defined: $\epsilon_\kappa = \epsilon/\epsilon_L$, $\gamma_\kappa = \gamma/\gamma_L$, $\rho_\kappa = \rho/\rho_L$, and $W_\kappa = W/W_L$. Using the closure relations and the dimensionless functions defined above, the resulting budget equation for the dimensionless energy flux for the case of a stationary $[\partial E_s(t,k)/\partial t = 0]$ turbulent suspension reads

$$\frac{d\epsilon_\kappa}{d\kappa} + C\frac{\epsilon_\kappa}{\kappa}T_\kappa + \frac{C_1}{\mathrm{Re}_s}\left(\frac{\kappa\epsilon_2^\kappa}{\rho_2^\kappa}\right)^{1/3}(1+T_\kappa) = W_\kappa, \tag{11}$$

where

$$T_\kappa = \frac{\phi\delta\gamma_\kappa}{(1+\phi)(1+2\delta\gamma_\kappa)+(\delta\gamma_\kappa)^2}, \tag{12}$$

and in which $C = C_1C_2$ and $\delta = \tau_p\gamma_L$ is the dimensionless particle response time. The suspension Reynolds number is defined by $\mathrm{Re}_s = Lv_L/\nu_L$. L is the integral length scale and v_L the integral velocity scale defined by $\nu_L = (\epsilon_L L/\rho_L)^{1/3}$. ν_L is the effective kinematic viscosity of the suspension for $k = L^{-1}$ and is given by $\nu_L = \nu(\rho_f/\rho_L)$ with ρ_L the effective density of the suspension for $k = L^{-1}$ given by $\rho_L = \rho_f[1+\phi(1+2\delta)/(1+\delta)^2]$. The fluid Reynolds number is defined by $\mathrm{Re}_f = Lv_L/\nu$ and is related to the suspension Reynolds number in the following way $\mathrm{Re}_s = \mathrm{Re}_f\nu/\nu_L$.The functions ρ_κ and γ_κ can be shown to be given by

$$\rho_\kappa = \left[1+\phi\frac{1+2\delta\gamma_\kappa}{(1+\delta\gamma_\kappa)^2}\right] \Bigg/ \left[1+\phi\frac{1+2\delta}{(1+\delta)^2}\right] \tag{13}$$

and

$$\gamma_\kappa = \frac{\kappa^2}{C_2\mathrm{Re}_s\rho_\kappa} + \frac{\epsilon_\kappa^{1/3}\kappa^{2/3}}{\rho_\kappa^{1/3}}. \tag{14}$$

L'vov *et al.* solve Eq. (11) together with Eqs. (12)–(14) as a function of the relevant dimensionless parameters: the particle mass fraction ϕ, the dimensionless particle response time δ, and the fluid Reynolds number Re_f. Also the particle volume fraction ψ is a dimensionless parameter. However, in the calculations the ratio of the particle density and fluid density is assumed to be large (for instance, a suspension of solid particles in gas), so that although the mass fraction was considerable the volume fraction was negligible. From the calculated spectrum for the energy flux the energy spectrum of the (stationary) suspension is determined using the closure relation.

As discussed in this publication we will investigate the behavior of a decaying $[W(t,k)=0]$ homogeneous, isotropic suspension. To that purpose we have to include the time-dependent term in the balance equation for the energy flux Eq. (11). Using the closure relations and dimensionless functions defined above we find now after some lengthy but straightforward calculations the following balance equation for the (dimensionless) energy flux:

$$f(\tau,\kappa)\frac{\partial\epsilon_\kappa(\tau,\kappa)}{\partial\tau} + \frac{\partial\epsilon_\kappa(\tau,\kappa)}{\partial\kappa} + g(\tau,\kappa) = 0, \tag{15}$$

where

$$f(\tau,\kappa) = \frac{1}{3}C_1\kappa^{-5/3}\rho_\kappa^{1/3}\epsilon_\kappa^{-1/3}\left[1 - \frac{1}{2}\frac{\epsilon_\kappa}{\rho_\kappa}\frac{(\partial\Phi/\partial\epsilon_\kappa)}{(\partial\Phi/\partial\rho_\kappa)}\right] \tag{16}$$

and

$$g(\tau,\kappa) = C\frac{\epsilon_\kappa}{\kappa}T_\kappa + \frac{C_1}{\mathrm{Re}_s}\left(\frac{\kappa{\epsilon_\kappa}^2}{\rho_\kappa}\right)^{1/3}(1+T_\kappa), \tag{17}$$

with

$$\frac{\partial\Phi}{\partial\epsilon_\kappa} = -C_3\phi\left[2\delta(1+\delta\gamma_\kappa)(2+3\delta\gamma_\kappa)\frac{\partial\gamma_\kappa}{\partial\epsilon_\kappa}\right] \tag{18}$$

and

$$\frac{\partial\Phi}{\partial\epsilon_\kappa} = 1 - C_3\phi\left[2\delta(1+\delta\gamma_\kappa)(2+3\delta\gamma_\kappa)\frac{\partial\gamma_\kappa}{\partial\rho_\kappa}\right] \tag{19}$$

in which

$$\frac{\partial\gamma_\kappa}{\partial\epsilon_\kappa} = \frac{1}{3}\frac{\kappa^{2/3}}{{\epsilon_\kappa}^{2/3}{\rho_\kappa}^{1/3}} \tag{20}$$

and

$$\frac{\partial\gamma_\kappa}{\partial\rho_\kappa} = -\frac{\kappa^2}{C_4{\rho_\kappa}^2} - \frac{1}{3}\frac{{\epsilon_\kappa}^{1/3}\kappa^{2/3}}{{\rho_\kappa}^{4/3}}. \tag{21}$$

The constants C_3 and C_4 are equal to $C_3 = [1+\phi(1+2\delta)/(1+\delta)^2]^{-1}$ and $C_4 = C_2\mathrm{Re}_s$. τ is the dimensionless time defined as $\tau = t/\tau_c$ with $\tau_c = L^{2/3}/(\epsilon L/\rho_f)^{1/3}$. The new budget equation (15) for the energy flux has some interesting features: the g term in the equation represents the energy dissipation due to the particle-fluid friction and the internal friction because of the fluid viscosity. The first two terms describe the influence of the cascade process of turbulence. When the g term is neglected Eq. (15) becomes a kind of wave equation in κ space with a time- and wave-number-dependent wave velocity $f(\tau,\kappa)^{-1}$.

In order to be able to solve Eq. (15) an initial condition and a boundary condition are needed. We will study that part of the spectrum that runs from $\kappa=1$ (the small-wave-number side of the inertial subrange) via the inertial subrange well into the dissipation range. We assume also that for $\tau<0$ the stirring force is still feeding turbulent energy at $\kappa=1$ into the inertial subrange. At $\tau=0$ the stirring force is stopped, the energy flux at $\kappa=1$ disappears and the decay process starts. So the boundary condition is, for $\tau\geqslant 0$, the energy flux $\epsilon_\kappa=0$ at $\kappa=1$. As initial condition ($\tau=0$) we choose for the energy flux the following spectrum for $\kappa\geqslant 1$:

$$\epsilon_\kappa=\left(1-\frac{C_1}{4\mathrm{Re}_f}\kappa^{4/3}\right)^3\bigg/\left(1-\frac{C_1}{4\mathrm{Re}_f}\right)^3. \tag{22}$$

For values of κ considerably smaller than $(4\mathrm{Re}_f/C_1)^{3/4}$ this spectrum is approximately equal to $\epsilon_\kappa=1$. So the (dimensionless) flux is for such κ values equal to its value at $\kappa=1$ (or $k=L^{-1}$).This behavior can be expected for stationary turbulence in the inertial subrange for the case without particles. For larger κ values the energy flux ϵ_κ decreases due to viscous dissipation. The spectrum of Eq. (22) is derived by L'vov *et al.* using certain approximations. We use this spectrum as our initial condition and calculate its development in time due to the decay process. According to the idea of universality of turbulence the properties of the energy flux though the energy spectrum become independent of the initial condition after a relaxation time. (This is due to the locality in the energy transfer in wave number space, i.e., eddies which effectively interact have similar wave numbers.) So we do not expect a strong dependence of our results on the initial condition.

After ϵ_κ has been calculated for a certain case from Eq. (15) the energy spectrum of the suspension can be determined using the closure relation

$$E_{s,\kappa}=\epsilon_\kappa{}^{2/3}\rho_\kappa{}^{1/3}\kappa^{-5/3}. \tag{23}$$

L'vov *et al.* have shown that the energy flux of the carrier fluid can be calculated from the suspension spectrum in the following manner:

$$E_{f,\kappa}=E_{s,\kappa}/\rho_\kappa=\epsilon_\kappa{}^{2/3}\rho_\kappa{}^{-2/3}\kappa^{-5/3}. \tag{24}$$

($E_{s,\kappa}$ and $E_{f,\kappa}$ have been made dimensionless by means of their values at $\kappa=1$.) In this way it becomes possible to study the decay of the turbulent energy spectrum of the fluid as a function of the relevant dimensionless groups, namely the particle mass fraction ϕ, the dimensionless particle response time δ, and the fluid Reynolds number Re_f. A computer program has been developed to carry out the calculations.

III. DNS RESULTS FOR A DECAYING TURBULENT SUSPENSION

Ferrante and Elghobashi [9] present a study to analyze their recent direct-numerical-simulation (DNS) results to explain in some detail the main physical mechanisms responsible for the modification of decaying homogeneous, isotropic turbulence by dispersed solid particles. In their study they fix both the volume fraction ($\psi=10^{-3}$) and mass fraction ($\phi=1$) for four different types of particles, classified by their ratio of the particle response time (τ_p) and the Kolmogorov time scale of turbulence (τ_κ). From the values of the volume fraction and mass fraction it follows, that the ratio of the particle density (ρ_p) and the fluid density (ρ_f) is 10^3. The ratio τ_p/τ_κ has the values 0.1, 0.25, 1.0, and 5.0. As the mass fraction is kept constant, the number of particles per unit of volume decreases in the numerical simulation with increasing particle response time. The total number of particles is considerable (80 million for a typical case). The particles are treated as point particles. Their simulations are carried out with and without including the effect of gravity. The numerical study has been performed with high resolution.

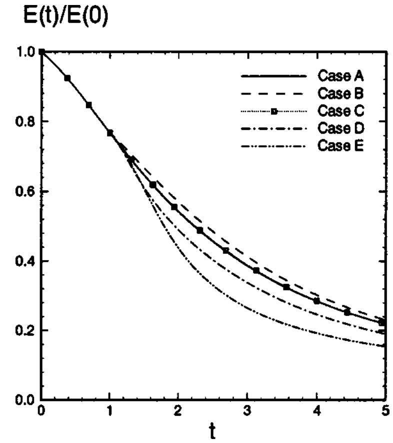

FIG. 1. Dimensionless turbulent kinetic energy as a function of dimensionless time. From Ferrante and Elghobashi [9].

In their publication Ferrante and Elghobashi discuss a number of interesting physical effects. Here we will concentrate on one particular effect and compare their results with our theoretical predictions for this effect. In Fig. 1 we show the result for the time evolution of the decaying turbulent kinetic energy of the carrier fluid $E(t)$, normalized by its initial value $E(0)$ at $t=0$, for the case without gravity. The particles are released in the turbulent flow field at $t=1$. [Their notation for the turbulent kinetic energy E is in our notation given by E^f. Their notation t represents time made dimensionless with $\tau_c=0.2144$ s. In our calculations we have made time dimensionless by means of $\tau_c=L^{2/3}/(\epsilon_L/\rho_f)^{1/3}$. So when we want to compare their numerical results with our predictions, we have to translate our dimensionless time τ to their t.] In Fig. 1 the result indicated by case A is for the particle-free flow, the results indicated by cases B, C, D, and E are for the carrier fluid in the suspension with particles of increasing response time ($\tau_p/\tau_\kappa=0.1$, 0.25, 1.0, and 5.0), respectively. It is clear that the smallest particles (with $\tau_p/\tau_\kappa=0.1$) reduce the decay rate of the (dimensionless) turbulent kinetic energy with respect to the particle-free flow, resulting in $E(t)/E(0)$ being larger than that for the particle-free flow. This is the particular effect, that we mentioned above. (Ferrante and Elghobashi call the particles of case B "microparticles.") The particles with a considerably larger inertia (cases D and E) initially enhance the decay rate of the turbulent kinetic energy resulting in values of the kinetic energy being smaller than for the particle-free flow at all times. The

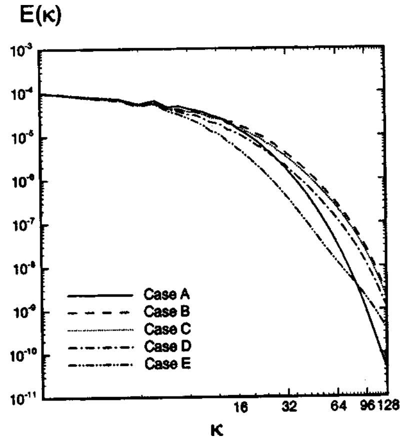

FIG. 2. Dimensionless kinetic energy of the carrier fluid as function of dimensionless wave number at $t=5.0$. From Ferrante and Elghobashi [9].

larger the particles (the larger their inertia and response time), the stronger the damping of the turbulence. After a certain period the difference between the turbulent kinetic energy for the suspension with the large particles and the turbulent kinetic energy for particle-free flow does not increase anymore. Ferrante and Elghobashi give particles for which the response time τ_p is equal to the Kolmogorov time τ_κ (case D) the name "critical particles." The still larger particles (case E) are called "large particles." There is a special case (case C), for which the damping rate is nearly the same as for the particle-free case. For this reason Ferrante and Elghobashi denote the particles of case C as "ghost particles," since their effect on the turbulence cannot be detected by their temporal behavior of the turbulent kinetic energy.

Figure 2 (Fig. 3 of Ferrante and Elghobashi) shows the energy spectra $E(t,\kappa)$ for the carrier fluid in the suspension for the five cases (A, B, C, D, and E) at dimensionless time $t=5$. Microparticles (case B) increase $E(t,\kappa)$ relative to the particle-free flow (case A) at wave numbers $\kappa \geqslant 12$, and reduce $E(\kappa)$ relative to case A for $\kappa < 12$, such that $E(t) = \int E(t,\kappa) d\kappa$ in case B is larger than in case A as shown in Fig. 1. Also for the cases C, D, and E the particles dampen the turbulence at small wave number compared to the particle-free flow and enhance the turbulence at high wave number. However the crossover wave number (the wave number where the influence of the particles changes from a turbulence-damping effect to a turbulence-enhancing one) increases with increasing particle response time. As can be seen from Fig. 2 large particles (case E) contribute to a faster decay of the turbulent kinetic energy by reducing the energy content at almost all wave numbers, except for $\kappa > 87$, where a slight increase of $E(t,\kappa)$ occurs.

We will now repeat briefly the explanation given by Ferrante and Elghobashi for the mechanisms responsible for the modification of decaying turbulence as shown in Figs. 1 and 2. We start with the microparticles. Because of their fast response to the turbulent velocity fluctuations of the carrier fluid, the microparticles are not ejected from the vortical structures of their initial surrounding fluid. The inertia of the microparticles causes their velocity autocorrelation to be larger than that of the surrounding fluid. Since the microparticles' trajectories are almost aligned with fluid points' trajectories, and their kinetic energy is larger than that of the surrounding fluid, the particles will transfer part of their own energy to the fluid. On the other hand, the microparticles increase the viscous dissipation rate relative to that of the particle-free flow. The reason is that the microparticles remain in their initially surrounding vortices, causing these vortical structures to retain their initial vorticity and strain rates longer than for the particle-free flow. The net effect is positive for the turbulent kinetic energy of the carrier fluid, as the gain in energy due to the transfer of energy from the particles is larger than the increase in viscous dissipation.

For large particles the explanation is different. Because of their significant response time large particles do not respond to the velocity fluctuations of the surrounding fluid as quickly as microparticles do, but rather escape from their initial surrounding fluid (crossing the trajectories of fluid points). Large particles retain their kinetic energy longer than the surrounding fluid. However, because of the "crossing trajectories" effect the fluid velocity autocorrelation is larger than the correlation between the particle velocity and the fluid velocity, causing a transfer of energy from the fluid to the particles. On the other hand, large particles reduce the lifetime of eddies, causing a viscous dissipation rate which is smaller than for the particle-free flow. The net result of the two opposing effects is a reduction of turbulent kinetic energy for a suspension with large particles at nearly all wave numbers relative to the kinetic energy for the particle-free turbulent flow.

It is emphasized that the explanation given above is a brief summary of the explanation given in the publication of Ferrante and Elghobashi. For more details their publication should be studied.

IV. THEORETICAL PREDICTIONS

As mentioned in the Introduction the idea of this publication is to compare predictions made with the theoretical model (extended for the application to a decaying turbulent suspension) with the DNS results of Ferrante and Elghobashi and to explain the results in terms of our model. To that purpose we have repeated with our model the Ferrante-Elghobashi calculations shown in Figs. 1 and 2. The results are given in Figs. 3–10. In Fig. 3 we first show the comparison between the time development of the turbulent kinetic energy of the carrier fluid (normalized by its initial value) for the particle-free flow as found from the numerical simulations and as predicted by the model. In order to compare our predictions with the DNS results we have made time dimensionless in Fig. 3 with $\tau_c = 0.2144$ s, as used by Ferrante and Elghobashi. As can be seen the agreement between model predictions and DNS results is reasonable.

Of course, much research has been carried out on the decay of a homogeneous, isotropic turbulent flow of a fluid without particles. For instance, in Hinze's book on turbulence [20] a review is given about this topic. It is stated that in the initial period of decay (when the inertial effects are important) the turbulent energy decreases with time as t^{-1}

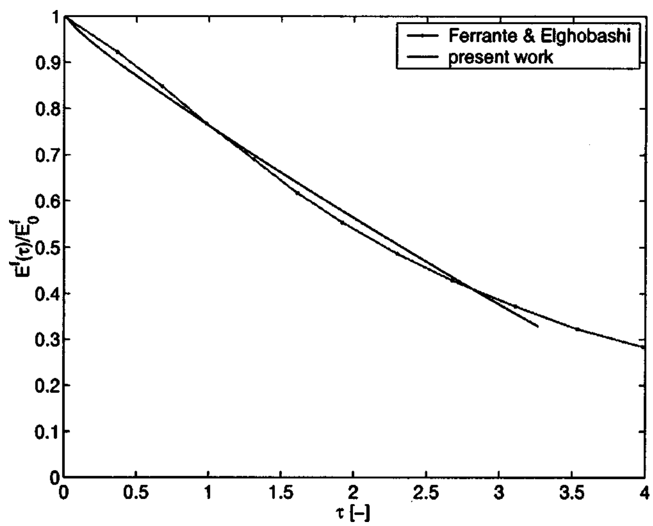

FIG. 3. Dimensionless turbulent kinetic energy of the fluid as function of dimensionless time for the particle-free case. Comparison between model predictions and DNS results.

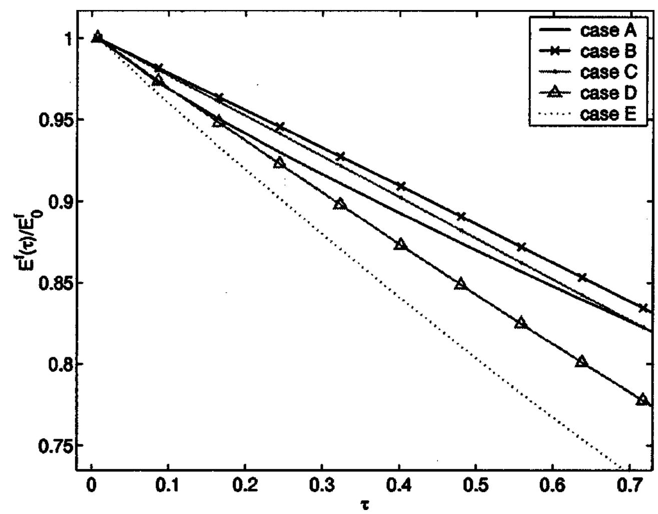

FIG. 5. Dimensionless turbulent kinetic energy of the carrier fluid in a suspension as a function of dimensionless time ($0<\tau<0.7$).

and in the final period (when viscosity effects dominate) the energy decreases as $t^{-5/2}$. In more recent work, see, for instance, Stalp, Skrbek, and Donally [21], it is reported that initially the energy decays as $t^{-6/5}$, then in case that the energy containing length scale saturates (because it reaches the size of the containing vessel) it decays as t^{-2}, and in the final period again as $t^{-5/2}$. In our future work we will make a detailed comparison between predictions made with our model and the results literature results mentioned above for a particle-free turbulent flow. In this study we were, in particular, interested in a comparison with the DNS results of Ferrante and Elghobashi. Our model predictions for the turbulent decay rate for the particle-free case agree reasonably well with their numerical results.

Figures 4 and 5 show model predictions for the time evolution of the decaying turbulent kinetic energy of the carrier fluid $E^f(\tau)$, normalized by its initial value, for suspensions with different particle response time (also the energy for the particle-free flow is shown). Similar to Ferrante and Elghobashi the volume fraction ($\psi=10^{-3}$) and mass fraction ($\phi=1$) are fixed and the four types of particles correspond to the following values of the ratio (τ_p/τ_κ): 0.1, 0.25, 1.0, and 5.0. So the results for cases A–E in Figs. 4 and 5 can be compared with those for cases A-E in Fig. 1. In Fig. 4 the time development of the turbulent kinetic energy is given for ($0<\tau<3.5$). To see the initial development in more detail we show in Fig. 5 the result in the smaller interval ($0<\tau<0.7$). It can be seen from the figures, that with increasing particle response time the turbulent energy of the carrier fluid decreases. The larger the particles (the larger their inertia and response time), the stronger the turbulence damping. However, as found in the DNS calculations, after a certain period the difference between the turbulent energy of

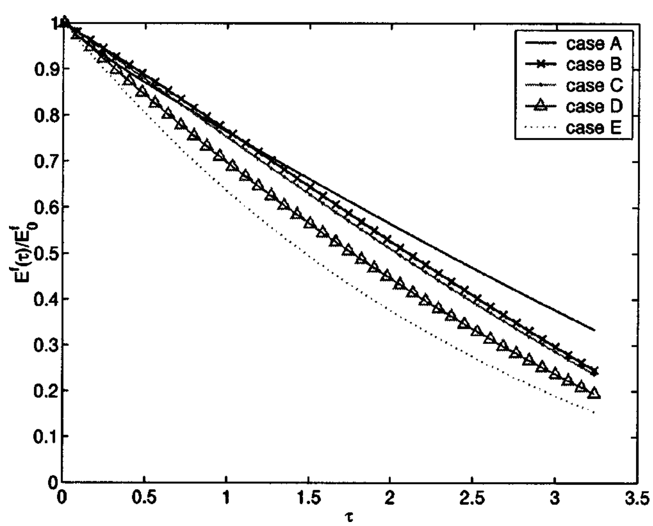

FIG. 4. Dimensionless turbulent kinetic energy of the carrier fluid in a suspension as a function of dimensionless time ($0<\tau<3.5$).

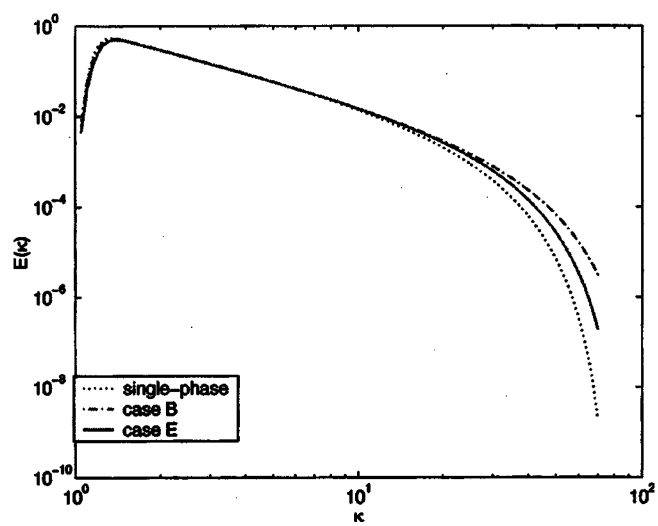

FIG. 6. Dimensionless turbulent kinetic energy of the carrier fluid as a function of dimensionless wave number ($\tau\approx0.5$).

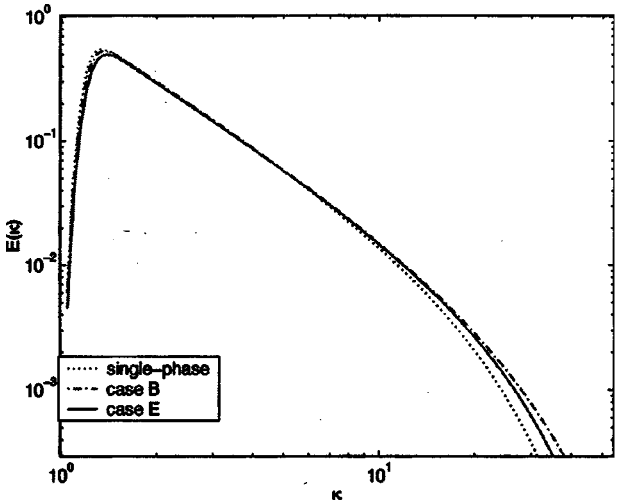

FIG. 7. Dimensionless turbulent kinetic energy of the carrier fluid as a function of dimensionless wave number ($\tau \approx 0.5$). (Enlargement of Fig. 6.)

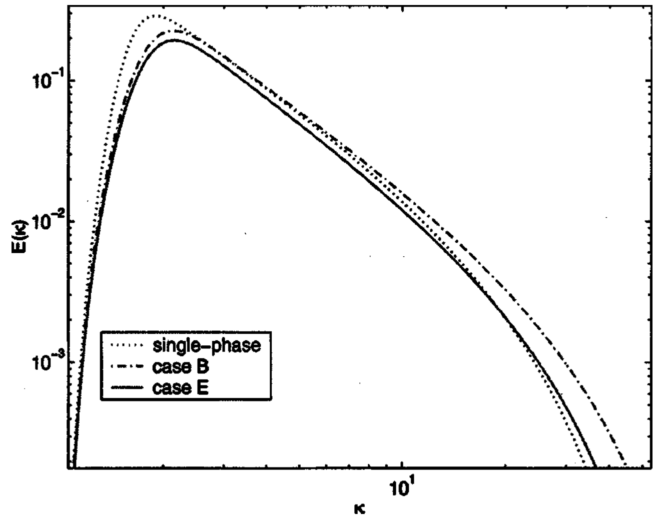

FIG. 9. Dimensionless turbulent kinetic energy of the carrier fluid as a function of dimensionless wave number ($\tau \approx 1.5$). (Enlargement of Fig. 8.)

the suspension for the large particles and the energy for the particle-free case does not seem to increase anymore. For the smallest particles ($\tau_p/\tau_\kappa = 0.1$) there is a reduction in the decay rate during the period ($0 < \tau < 1$). Also the DNS results show this behavior (see Fig. 1), but during a larger period ($1 < \tau < 5$). So qualitatively the theoretical predictions agree well with the DNS results, although quantitatively there are some differences. We will come back to this point later on.

We have also calculated the energy spectra $E^f(\tau,\kappa)$ for the carrier fluid in the suspension for the five cases (A, B, C, D, and E) and compared the predictions with the DNS results given in Fig. 2. We will only show the results for the particle-free flow (case A), the microparticles (case B), and the large particles (case E). The results for ghost particles (case C) and critical particles (case D) are in between those for cases B and E. For $\tau \approx 0.5$ the results are shown in Fig. 6 (with an enlargement in Fig. 7), for $\tau \approx 1.5$ in Fig. 8 (with an enlargement in Fig. 9), and for $\tau \approx 3.0$ in Fig. 10.

There is a difference between our results and the DNS results at small values of κ ($\kappa \approx 1$). As discussed we assume that at $\tau = 0$ the stirring force is stopped, the energy flux at $\kappa = 1$ disappears and the decay process starts. So for $\tau \geqslant 0$ the energy flux $\epsilon_\kappa = 0$ at $\kappa = 1$. Due to the cascade process of turbulence the area where $E^f(\tau,\kappa)$ is influenced by the boundary condition at $\kappa = 1$, grows towards larger κ values with increasing time (see the development of the spectrum from $\tau \approx 0.5$, via $\tau \approx 1.5$ to $\tau \approx 3.0$ in Figs. 6–10). In the DNS calculations an initial spectrum is selected with $E^f(\tau,\kappa) = 0$ for $\kappa = 0$. At $\tau = 0$ the stirring force is stopped and the spectrum starts to decay. As can be seen from Fig. 2 this leads to a different boundary condition at $\kappa = 1$. There is obviously still an energy input into the spectrum at $\kappa = 1$ for $\tau > 0$ from the larger eddies. This may explain the detailed differences between model predictions and DNS results. However, as discussed the main conclusions are in our opinion independent of the precise formulation of the initial condition.

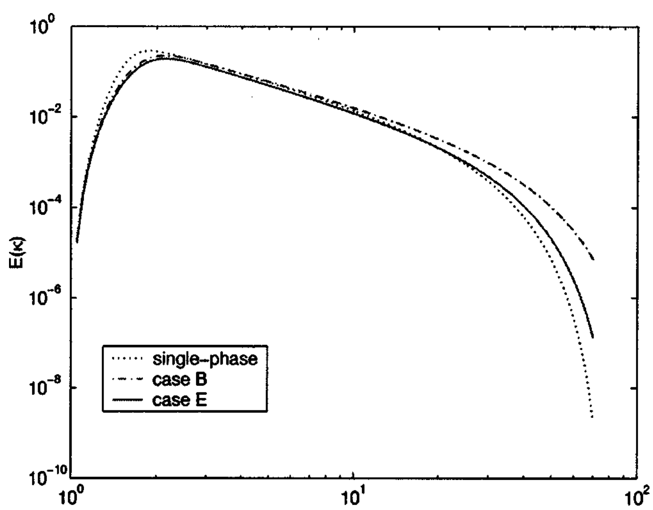

FIG. 8. Dimensionless turbulent kinetic energy of the carrier fluid as a function of dimensionless wave number ($\tau \approx 1.5$).

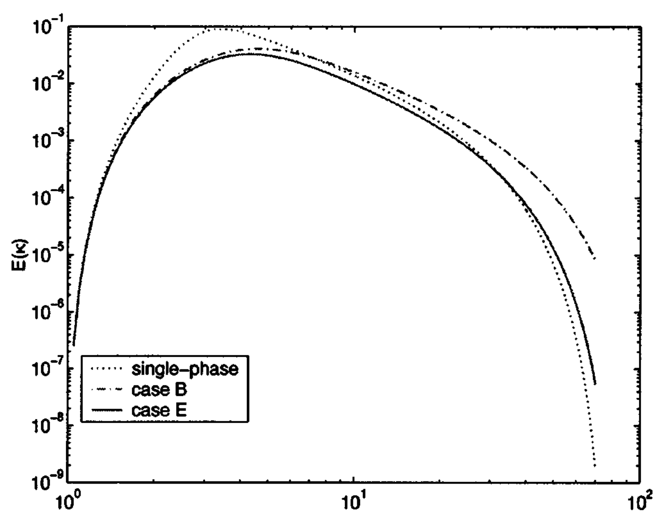

FIG. 10. Dimensionless turbulent kinetic energy of the carrier fluid as a function of dimensionless wave number ($\tau \approx 3.0$).

It is clear from Figs. 6–10 that the particles dampen the turbulence for small values of κ (large eddies) and enhance the turbulence for large values of κ (small eddies). However, there is a difference. The microparticles (case B) enhance the turbulence over a much larger range of κ-values than the large particles (case E). For microparticles the enhancement is so strong, that for $0<\tau<1$ the total energy over all eddies is larger than for the particle-free flow. That is not the case for the large particles. The crossover wave number (the wave number where the influence of the particles changes from a turbulence-damping effect to a turbulence-enhancing one) increases with increasing particle response time. This result is the same as found in the DNS calculations and as shown in Fig. 2.

We will now give an explanation of the observed phenomena in Figs. 6–10 in terms of our theoretical model. In principle the explanation is similar to the one given for the case of a stationary turbulent suspension (see L'vov *et al.*). An important effect of the particles is that they increase the effective density of the suspension. As the dynamic viscosity is not much influenced at low values of the particle volume fraction, the kinematic viscosity of the suspension will decrease compared to the kinematic viscosity for the particle-free case. This will decrease the Kolmogorov length scale and hence elongate the inertial subrange of the energy spectrum. Mathematically this can be seen in the following way. For instance, for particles with a very small response time (τ_p) Eq. (15) reduces to the equation for the particle-free flow apart from the fact that the fluid Reynolds number Re_f is replaced by the suspension Reynolds number Re_s. For small particle response time $\mathrm{Re}_s \approx \mathrm{Re}_f(1+\phi)$, so $\mathrm{Re}_s > \mathrm{Re}_f$. This means that the viscous damping term $g(\tau,\kappa)$ in Eq. (15) is smaller than for the particle-free flow and the Kolmogorov wave number shifts towards larger κ values.

There is a second effect, that is in particular important in the inertial subrange. There are two competing effects in that subrange: an energy suppression due to the fluid-particle friction and an energy enhancement during the cascade process due to the decrease of the effective density of the suspension with decreasing eddy size. Particles become less involved in the eddy motion with decreasing eddy size. A more detailed investigation of this effect has been made by L'vov *et al.* and it is shown that this effect can lead to a significant enhancement of the turbulence in the inertial subrange dependent on the conditions such as the ratio of the particle response time and the integral time scale. It is the combination of the two effects mentioned above, that explains the phenomena observed in Figs. 6–10 in terms of our model.

V. DISCUSSION

An interesting conclusion of this work is that it seems possible to give two different physical explanations for the influence of particles on a (decaying) homogeneous, isotropic turbulent suspension. One explanation (given by Ferrante and Elghobashi) is based on a "microscopic" picture about the interaction between individual particles and their local fluid flow environment. The other one (given in this publication and earlier by L'vov *et al.*) uses a "macroscopic" picture with eddy-size-dependent suspension properties, such as effective density, effective viscosity, effective damping, etc. Both pictures give a satisfactory explanation, not only in words but also mathematically.

ACKNOWLEDGMENTS

The authors acknowledge the stimulating discussions with Professor Dr. V. S. L'vov and Dr. A. Pomyalov of the Weizmann Institute in Israel. They are also grateful to Dr. ir. M. J. B. M. Pourquié for his support with the numerical calculations.

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